Private equity investors have a vast universe of investment opportunities – from angel financing to leveraged buyouts. They can invest in primary funds, buy secondary interests in existing funds or invest directly into companies. They can build up global programmes or focus on certain regions. Since the universe of investment opportunities is so diverse, the asset allocation has a profound impact both on the risk and the return of a private equity portfolio.

More than fifty years ago, Harry Markowitz, Nobel Prize co-recipient for ‘Modern Portfolio Theory and the Capital Asset Pricing Model’, proposed the mean-variance framework for portfolio optimisation, which is considered to be a major milestone of modern finance theory. In this model, the optimal portfolio maximises the expected return for a given risk measured through volatility.

While providing the first quantitative asset allocation framework, the resulting optimal portfolios are subject to different drawbacks. First, they are generally concentrated in a small number of assets; and second, the optimal solution is highly sensitive to changes of the input parameters.

There is an additional shortcoming: private equity does not allow for portfolios to be rebalanced in a cheap and simple way. Therefore, the asset allocation process that optimises the portfolio has to be stable over time, in the sense that two successive optimal allocations should not differ drastically. This property if often referred to as robustness.

### RISK AND ROBUSTNESS

During recent years, asset allocation models that focus on risk and disregard returns have attracted great interest, since they are more robust than traditional models. Estimating returns from past samples can lead to erratic results – whereas focusing on risk generally produces more stable models.

Additionally, empirical evidence shows that periods of low volatility generally coincide with periods of high returns. For example, in a study on the relationship between volatility and return of the S&P 500 index, Crestmont Research shows that lower volatility corresponds to a higher probability of positive returns.

Private equity on average outperforms public equity. Between 1981 and 2011, private equity returned on average 13.02 percent (source: Thomson One) while the public market returned 9.87 percent (source: Bloomberg, MSCI Daily TR Gross World). However, fund performance is very diverse; in order to achieve superior returns, an investor has to be able to select first class managers. This can be achieved by a rigorous selection procedure.

One possible solution is a two-step investment process: the first consists in defining an asset allocation with the objective of minimising risks and maximising diversification; the second consists of maximising the return by means of superior fund selection. While both steps are equally important, this paper focuses solely on the first step.

### 1. The risk-weighted asset allocation

Risk can be defined in numerous ways. The simplest method is to use volatility within a certain window; unfortunately, the size of the window heavily impacts the results. For the purposes of this article, we will work with a definition of risk that does not depend upon the choice of a window but that gives more weights to recent events: the RiskMetrics model (although we modified it to be more sensitive to negative shocks (losses) than positive shocks (gains) in a way that is analogous to the semi-volatility or downside volatility).

For many of the traditional asset classes, asset allocation is generally tailored to maximise returns while limiting risks. However, in the context of private equity, trying to maximise returns via asset allocation does not make sense. The large spread in performance between different funds (see Figure 1) implies that the quest for return...
takes place in the selection process rather than via asset allocation.

Figure 1 (p. 54) shows the performance of various private equity asset classes – US buyout, US venture capital, EU buyout and EU venture capital, which represent the major part of the global private equity universe and offer a long history, with quarterly indices calculated by Thomson One. The dispersion between the upper quartile and the lower quartile is close to 15 percent.

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Equal risk contribution portfolios (ERC) try to maximise diversification in terms of risk: each asset class contributes the same amount of risk to the portfolio (Sébastien Maillard, Thierry Roncalli and Jérome Teiletche; ‘On the Properties of Equally-Weighted Risk Contributions Portfolios’, 2008). This means that an asset class with a higher risk will obtain a lower allocation than an asset class with a lower risk. The calculation of the allocation for the ERC portfolio is based on the risk of each asset class and on the correlation between the asset classes. Equal risk contribution portfolios are often compared to equally weighted portfolios (EW), which allocate the same weight to each asset. Figure 2 (p. 55) compares both portfolio types in terms of their allocations and risk contributions.

**Figure 2: Risks and weights**

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2. **Market-weighted risk contribution model**

In private equity, the different asset classes represent different geographies, strategies and investment styles. And they are not comparable in terms of market sizes: for example, the US buyout market is much bigger than the European venture capital market. Instead of allocating the portfolio in such a way that each asset class contributes the same amount of risk, we allocate the risk according to the market weight of each asset class. We call the resulting portfolios market-weighted risk contribution portfolio (MWRC). In a market-weighted risk contribution portfolio, an asset class with a larger market will have a larger allocation. Additionally, a higher risk in an asset class will lead to a reduction of the allocation, similar to ERC portfolios.

The market weight used can, for example, correspond to the total size of each asset class at a given date (see Table 1, p. 55). Alternatively, one can go beyond simple market capitalisation and adapt the weights to an investor’s view of the investable market – for example, the market weights could be determined only from those funds deemed of sufficient quality for institutional investors.

In order to assess the soundness of the MWRC model, we are going to compare it to the equal weighted portfolio (EW) model, which allocates uniformly across all asset classes. Such an allocation is often used by investors who want to maximise the diversification. As discussed above, EW portfolios may lead to concentrations in terms of risk. Additionally, we consider minimum variance portfolios (MV) – whose only objective is to lower risk, rather than aiming to optimise the risk/return ratio. Similar to MWRC portfolios, the

**Table 1: Asset classes by size**

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Market Capitalisation (USD billion)</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>US BO</td>
<td>813</td>
<td>55.3%</td>
</tr>
<tr>
<td>US VC</td>
<td>256</td>
<td>17.4%</td>
</tr>
<tr>
<td>EU BO</td>
<td>346</td>
<td>23.5%</td>
</tr>
<tr>
<td>EU VC</td>
<td>56</td>
<td>3.8%</td>
</tr>
</tbody>
</table>

Source: Thomson One, as of Q2 2012
only required optimisation inputs are correlations and volatilities.

Figure 3 (p. 56) compares the results obtained by the different asset allocation models. Minimum variance portfolios suffer from the same drawbacks as other optimal portfolios in the mean-variance framework: they are concentrated in a small number of assets. For example, from 2008 to 2012, the optimal portfolio contains only two assets out of four. Further, the optimal solution is highly sensitive to changes of the input parameters – so from one quarter to the next, large changes in the asset allocation may take place.

Once the asset allocation has been calculated in theory, it needs to be implemented in practice. In private equity, allocations are built over time by commitments to funds. Selling fund interests on the secondary market can also be an option, but may lead to substantial losses. Therefore we assume that an investor commits to new funds based on the calculated asset allocation. For example, the asset allocation computed for Q4 2006 has been used to allocate commitments to private equity funds in 2007.

We have simulated the effect of such a commitment program using aggregate cashflow data from the Thomson One database for each asset class, starting in Q1 2000. Figure 4 (p. 56) presents the resulting allocations based on the net asset value for the different models.

In the case of MV, the allocation changes are abrupt and cannot be reproduced in reality, which leads to an allocation that is substantially different from the target allocation. The allocation based on the EW model leads to allocations that are highly stable, and only changing due to different returns in the different asset classes. MWRC gives allocations whose successive changes are smooth. Such an allocation can be implemented solely by means of new commitments.

If we look at the return of the different allocations from 2000 until today (listed in Table 2, p. 56), we see that the MWRC allocation delivered the best performance, while the volatility is considerably lower than volatility of the MV allocation. This shows that controlling the risks does not come with a loss of performance but might even enhance it, as often low volatility is associated with higher returns.

**CONCLUSION**

Market weighted risk contribution portfolios offer a sound asset allocation framework for private equity. The robustness of the solution allows investors to move from a theoretical setting to practical investment plans by committing accordingly. And the methodology enables them to adapt existing portfolios to the market environment by incorporating risk facts and information about the size of the investable market.

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